On the color simple group from chiral electroweak gauge groups

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Abstract

Following the pioneering Okubo scrutiny of gauge simple groups for the quantum chromodynamics we show the constraints coming from the wondrous predictive leptoquark-bilepton flavordynamics connecting the number of color charges to solution of the flavor question and to an electric charge quantization unconstrained from the Dirac, Majorana or Dirac–Majorana character of massive neutrino.

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Nobody has put to the test the interplay between the Fermi-TeV-kTeV [1] and the end of space-time Planck scales but everybody believes that there is more physics beyond the standard QCD and QFD. Since the standard model [2] of nuclear and electromagnetic interactions is very well confirmed [3,4] up to the TeV scale the possibilities for chiral gauge semisimple group extensions have not been exhausted yet. The most general chiral gauge semisimple group, expanding the number of color charges (n_c) and the weak isospin group (m) is

$$G_{n_c m_L 1_N} \equiv \mathrm{SU}(n_c)_c \otimes \mathrm{SU}(m)_L \otimes \mathrm{U}(1)_N$$

where the minimal extension is the G_{331} gauge symmetry [5,6]. Although the data accumulated on the scaling violations in deep inelastic scattering experiments are consistent with SU(3) gauge structure of strong interactions, extensions of the color sector [7] in which quarks transform under the fundamental representations of $\mathrm{SU}(n_c)_c$, $n_c = 4, 5$ were considered in the context of the electroweak standard model preserving the consistency at low energies [8]. However, for the first time too many fundamental questions of physics are answered and within the minimal semisimple gauge group extension of the standard model. Although the weak isospin group is minimally enlarged to $\mathrm{SU}(3)_L$ preserving the color and Abelian group factors there is a first unusual capacity in answering fundamental questions only by exploring the minimal enlargement of the electroweak gauge group. Consider, for instance, the following:

1. Each generation is anomalous and the anomalies cancel when the number of leptonic generations is divisible by the number of colors [5,6,9,10]. There is a relation between the strong and electroweak sectors of the model which does not exist in the standard model with the solution for the flavor question;

2. The electroweak mixing angle, θ_{W}, is limited from above with an upper bound determined by their Landau pole [11];

3. The neutrino and the charged leptons masses are constrained in the cubic seesaw relation

$$m_{\nu_\ell} \propto \frac{m_\ell^3}{M_W^2}, \quad \ell = e, \mu, \tau$$

   with outcomes for the solar neutrino problem and hot dark matter [12];

4. The Yukawa couplings have a Peccei–Quinn [13] symmetry which can be extended to all sectors of the Lagrangian with an invisible axion solving the strong-CP problem [14];

5. Spontaneous CP violation in the electroweak sector [15]. There are several natural sources of explicit and spontaneous CP violations [16];

6. The quark mass hierarchy [17];

7. Although the leptoquark-bilepton models do not conserve each generation lepton number L_ℓ, the neutrinoless double beta decay is forbidden because of the conservation of the quantum number $\mathcal{F} \equiv L + B$, where B is the barion number and $L = \sum_\ell L_\ell$ is the total lepton number. If this global symmetry is explicitly violated in the Higgs potential, there are contributions to the decay which depend less on the neutrino mass

than they do in too many extensions of the standard model [18]. The double beta decay with Majoron emission is possible as well [19];

8. There is an electric charge quantization without any constraint on the Dirac, Majorana or Dirac–Majorana [20] character of the massive neutral fermions [21].

Representation contents are determined by embedding the electric charge operator

$$\frac{\mathcal{Q}}{|e|} = (\Lambda_3 + \xi\Lambda_8 + \zeta\Lambda_{15}) + N \tag{1}$$

in the neutral generators $\Lambda_{3,8,15} = \lambda^{\mathrm{SU}(4)}_{3,8,15}/2$ of the largest weak isospin group SU(4) extension and N is the new $\mathrm{U}(1)_N$ charge equivalent to the electric charge average of the fermions contained in each flavor multiplet. If we consider the lightest leptons as the fermions which determine the approximate symmetry, and also independent flavor generations, then SU(4)×U(1) is the largest non-symmetric gauge group of the electroweak sector. There is no room for the chiral semisimple group SU(5)×U(1) if lepton electric charges are only 0, ±1. The weak hypercharge of the G_{321} standard model is

$$\frac{Y}{2} = (\xi\Lambda_8 + \zeta\Lambda_{15}) + N \tag{2}$$

and in the minimal G_{331} leptoquark-bilepton model, $\xi = -\sqrt{3}$, $\zeta = 0$, are contained 17 gauge vector fields,

$$\begin{aligned} \mathrm{SU}(3)_c : g^i_\mu &\sim (\mathbf{8}, \mathbf{1}, N = 0); \quad i = 1, 2, ..., 8; \\ \mathrm{SU}(3)_L : W^j_\mu &\sim (\mathbf{1}, \mathbf{8}, 0); \quad j = 1, 2, ..., 8; \\ \mathrm{U}(1)_N : B_\mu &\sim (\mathbf{1}, \mathbf{1}, 0), \end{aligned} \tag{3}$$

nine lepton fields connected through charge conjugation of the charged fields in three triplets,

$$L_\ell \sim (\mathbf{1}, \mathbf{3}, 0), \quad \ell = e, \mu, \tau; \tag{4}$$

three families of quarks,

$$\begin{aligned} Q_{1L} &\sim (\mathbf{3}, \mathbf{3}, +2/3) \\ u_R &\sim (\mathbf{3}, \mathbf{1}, +2/3) \\ d_R &\sim (\mathbf{3}, \mathbf{1}, -1/3) \\ J_{1R} &\sim (\mathbf{3}, \mathbf{1}, +5/3) \end{aligned} \tag{5}$$

for the first family, and

$$\begin{aligned} Q_{\alpha L} &\sim (\mathbf{3}, \bar{\mathbf{3}}, -1/3) \\ c_{\alpha R} &\sim (\mathbf{3}, \mathbf{1}, +2/3) \\ s_{\alpha R} &\sim (\mathbf{3}, \mathbf{1}, -1/3) \\ J_{\alpha R} &\sim (\mathbf{3}, \mathbf{1}, -4/3) \end{aligned} \tag{6}$$

where $\alpha = 2, 3$ labels the second and third families. Taking into account three color charges we have an amount of 54 quark fields. The J_1 and J_α leptoquark fermions are color-triplet particles with electric charge $\pm\frac{5}{3}$ and $\mp\frac{4}{3}$ which carry baryon number and lepton number, $B_{J_{1,\alpha}} = +\frac{1}{3}$, and $L_{J_\alpha} = -L_{J_1} = +2$. All masses are generated with four multiplets of scalar fields

$$\begin{aligned}\eta &\sim (\mathbf{1}, \mathbf{3}, 0)\\ \rho &\sim (\mathbf{1}, \mathbf{3}, +1)\\ \chi &\sim (\mathbf{1}, \mathbf{3}, -1)\\ S_{ij} &\sim (\mathbf{1}, \bar{\mathbf{6}}_S, 0)\end{aligned} \tag{7}$$

and in the symmetric phase of the theory they are parametrized by 30 real scalar fields. Such unavoidable scalarland is the most desirable field sector to have an experimental comprovation, since in theories with spontaneous symmetry breaking of the gauge symmetry it is essential but the unique field of the standard model which does not present evidences is the Higgs scalar boson. The total number of massless fields in the G_{331} model is 110 and there are not spin-$\frac{3}{2}$ Rarita–Schwinger fields.

Our main purpose is to select the possible color gauge simple groups from the leptoquark-bilepton flavordynamics. Let us remark that in a theory whith the $\mathrm{SU}(n_c)_c$ gauge simple group the 't Hooft [22] limit $n_c \to \infty$ and the Maldacena [23] conjecture provide the evidence of the gauge to string theories limit. In the four-dimensional super Yang–Mills type IIB string theory arises the color confinement and a mass gap within the 5-brane of the eleven-dimensional M-theory [24]. The $n_c = 3$ standard QCD is an asymptotically free theory including its non-perturbative confinement property. The perturbative strong coupling constant is

$$\alpha_{\mathrm{s}}(q) = \frac{g^2_{\mathrm{SU}(n_c)_c}(q)}{4\pi} = 4\pi\left[\beta_0 \ln\left(\frac{q}{\Lambda_{\mathrm{QCD}}}\right)^2\right]^{-1} \tag{8}$$

with

$$\beta_0 = \frac{11}{3}n_c - \frac{2}{3}n_f \tag{9}$$

and the fundamental scale $\Lambda_{\mathrm{QCD}} \simeq 250$ MeV $\simeq 10^{-3}(\sqrt{2}G_{\mathrm{F}})^{-\frac{1}{2}} \simeq 246 \times 10^{-3}$ GeV $= 10^{-3}\Lambda_{\mathrm{QFD}}$ where quarks form the hadrons as a direct effect of the color confinement and n_f is the number of quark flavors. The Λ_{QFD} is the Fermi scale of electroweak spontaneous symmetry breaking $G_{321} \to \mathrm{SU}(3)_c \times \mathrm{U}(1)_{\mathrm{em}}$. There are two limits,

$$\lim_{n_c\to\infty} \alpha_{\mathrm{s}}(q) = 0, \quad \lim_{q\to\infty} \alpha_{\mathrm{s}}(q) = 0. \tag{10}$$

At high energy, $q^2/\Lambda^2_{\mathrm{QCD}} \gg 1$, the strong coupling constant is small and the QCD is described by the perturbation theory. In 't Hooft original expansion the number of flavors is kept fixed when $n_c \to \infty$. The $\mathrm{SU}(n_c)_c$ exact symmetry is realized in the hidden way. In the weak coupling limit

$$a^2\Lambda^2_{\mathrm{QCD}} = \exp\left\{-\frac{1}{\beta_0}\left(\frac{4\pi}{g_{\mathrm{SU}(n_c)_c}(a)}\right)^2\right\} \tag{11}$$

where a could be the spacing scale of a lattice gauge theory of strong couplings the Maldacena conjecture provides a special evidence that a string theory comes out from a gauge theory [25]. The G_{331} theory has two anomalies containing the color gauge group. Characterizing each triangle anomaly by three generators associated to the gauge group they are $[\mathrm{SU}(3)_c]^3$ and $[\mathrm{SU}(3)_c]^2[\mathrm{U}(1)_N]$. The pure cubic color anomaly cancel since the QCD has a vector-like fermion representation content so there is independent anomaly cancellation in each color triplet and the associated antitriplets of quarks. Setting the notation for the standard quark chiral flavors

$$N_{u_R} = N_{c_R} = N_{t_R} \equiv N_{U_R}\,, \tag{12a}$$

$$N_{d_R} = N_{s_R} = N_{b_R} \equiv N_{D_R}\,, \tag{12b}$$

and

$$N_{Q_{2L}} = N_{Q_{3L}} \equiv N_{Q_{\alpha L}}, \tag{12c}$$

$$N_{J_{2R}} = N_{J_{3R}} \equiv N_{J_{\alpha R}} \tag{12d}$$

also for the leptoquark flavors, the $\mathrm{Tr}([\mathrm{SU}(3)_c]^2[\mathrm{U}(1)_N]) = 0$ constraint is

$$3(N_{Q_{1L}} + 2N_{Q_{\alpha L}}) - 3(N_{U_R} + N_{D_R}) - N_{J_{1R}} - 2N_{J_{\alpha R}} = 0 \tag{13}$$

and since the $\sum_{L_\ell} N_{L_\ell}$ term vanishes coincides with the mixed gravitational-gauge anomaly constraint $\mathrm{Tr}([\mathrm{graviton}]^2[\mathrm{U}(1)_N])= 0$. Also the $N_{Q_{1L}} + 2N_{Q_{\alpha L}}$ term vanishes in the minimal and extended leptoquark-bilepton models [5,6,26].

Being N_ℓ and N_q the number of lepton and quark generations let us consider the $\mathrm{SU}(n_c)_c$ possibilities for $n_c \geq 3$ where the $\mathrm{N}_\ell = \mathrm{N}_\mathrm{q} = \mathrm{N}_\mathrm{generations}$ coincidence is evaded. Denoting as $n_\mathbf{m}$ and $n_{\bar{\mathbf{m}}}$ the number of quark generation multiplets transforming as $\mathbf{m}$ and $\bar{\mathbf{m}}$ in the fundamental representation under the $\mathrm{SU}(m)_L$ flavor group factor we have the universality breaking condition in the lepton sector

$$\mathrm{N}_\ell = |n_c\,(n_\mathbf{m} - n_{\bar{\mathbf{m}}})|, \tag{14a}$$

and

$$\mathrm{N}_\mathrm{q} = n_\mathbf{m} + n_{\bar{\mathbf{m}}} \tag{14b}$$

for the number of quark flavor generations. The condition in Eq. (14a) involves the following possibilities: (1) $n_\mathbf{m} > n_{\bar{\mathbf{m}}}$, when the leptons must transform as $\bar{\mathbf{m}}$; (2) $n_\mathbf{m} < n_{\bar{\mathbf{m}}}$ when the lepton multiplets are attributed to the $\mathbf{m}$ representation; (3) $n_\mathbf{m} = n_{\bar{\mathbf{m}}}$. For $n_\mathbf{m} > n_{\bar{\mathbf{m}}}$ and in the case of even n_c, $n_c = 2k$, $k \geq 2$ we have the ratio

$$\frac{n_\mathbf{m}}{n_{\bar{\mathbf{m}}}} = \frac{2k+1}{2k-1} \tag{15}$$

but for odd $n_c = 2k + 1$ the ratio is

$$\frac{n_\mathbf{m}}{n_{\bar{\mathbf{m}}}} = 1 + \frac{1}{k}. \tag{16}$$

The $SU(5)_c$ group consistent with the standard flavordynamics [8] satisfies the $N_\ell = N_q$ condition if $n_{\mathbf{m}}/n_{\bar{\mathbf{m}}} = 3/2$ for $k = 2$ with five generations in a universal representation content. The ratios of the lepton and quark generations number with the number of color charges are

$$\frac{N_\ell}{n_c} = \frac{N_q}{n_c} = \frac{N_{\text{generations}}}{n_c} = k, \tag{17}$$

for all $k \in \{1, 2, 3, ...\}$ so as to

$$\lim_{n_c \to \infty} \frac{N_{\text{generations}}}{n_c} = 0 \tag{18}$$

for a finite k.

When $k \to \infty$ then $N_{\text{generations}} \to \infty$ for $n_c = 3$ or $n_c = 4, 5$ [8] but when $n_c \to \infty$ the $\frac{\infty}{\infty}$ indetermination arises. This could be seen as an intrinsic limitation of the theory with conformal invariance due to the horizontal replication of fundamental matter fields pointing from the particle to the string elementarity level.

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